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Experiments on natural convection in complex enclosed spaces containing either two fluids or a single fluid

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Abstract—Heat transfer coefficients were determined experimentally for natural convection in the enclosed space between two concentrically positioned vertical cylinders having different finite heights. The space was filled with a pair of fluid layers, one atop the other. The investigated two-fluid systems included air and water, air and hexadecane, and hexadecane and water. Experiments were also performed for the case in which the intercylinder space was filled with a single fluid—air, water, or hexadecane. Parametric variations were carried out for the height of the fluid–fluid interface and for the Rayleigh number. Comparisons were made between the experimental results and predictions obtained from numerical solutions of the problem. Very good agreement prevailed for the single-fluid cases and also for the two-fluid cases where meniscus and interfacial tension effects were either negligible or small. When these effects were more significant, as for the hexadecane–water system (the only liquid–liquid system investigated), it was found possible to achieve satisfactory agreement by making use of a relatively simple interface model. For the single-fluid results, the effect of the Prandtl number variation (between 0.71 and 40) was highlighted.

INTRODUCTION

THIS IS the second and concluding part of a two-part study of two-fluid natural convection in the enclosed space between two concentrically positioned vertical cylinders having different finite heights. The enclosed space is filled by a system of two fluids, one situated above the other in a layered manner. The first part of the study was concerned with numerical solutions of the problem, and an extensive presentation of results is set forth in ref. [1] along with a description of the models for which the computations were performed.

This paper is the experimental counterpart of ref. [1], the separate presentation of the experimental work having been necessitated by the unacceptably long length of a combined paper. As noted shortly, experimental results will also be presented for the case in which the intercylinder space is filled with a single fluid—specifically, each of the fluids which participated in the investigated two-fluid systems.

The two cylinders shared a common vertical axis, and the inner cylinder was nominally centered in the height of the outer cylinder. The inner cylinder was characterized by height H and diameter D, while the corresponding quantities for the outer cylinder were H_o and D_o. For the present experiments, $H = D$, $H_o = D_o$, and $D/D_o = 0.22$.

The experiments were performed utilizing three two-fluid systems : air–water, air–hexadecane, and hexadecane–water, with air, air, and hexadecane being the upper fluid in the respective systems and water, hexadecane, and water being the lower fluid. Single-fluid experiments were carried out with air, water, and hexadecane. The surfaces of the inner and outer cylinders, which served respectively as the inner and outer boundaries of the fluid-filled space, were maintained at uniform but different temperatures, with higher values at the inner cylinder.

Two parameters were varied systematically during the course of the experiments. One of these was the position of the interface between the two fluids. Consideration was given here to interfaces situated at various positions along the height of the inner cylinder. In particular, if z^* denotes the elevation of the interface above the base of the inner cylinder, then the overall range of investigated interface positions extended from about $z^*/H = 0.14$ to 0.91. The other parameter was the temperature difference between the inner and outer cylinders. Its variation resulted in a 20- to 50-fold variation of the Rayleigh number, depending on the particular fluid system.

The experimental data for both the two-fluid and single-fluid systems will be reported in terms of the average Nusselt number for the inner cylinder. Different definitions of the Nusselt number will be used to reflect the specifics of the heat flow paths for the various fluid configurations. Comparisons between the experimentally and numerically determined Nusselt numbers will be made for all cases. For the two-fluid systems, another issue to be considered in the presentation of results is the effect of the interfacial tension between the fluids, with particular reference to the investigated liquid–liquid system (hexadecane–water). According to the numerical results of ref. [1], only the liquid–liquid system was sensitive to the interfacial tension. For the single-fluid results, a special focus will be the correlation of the effect of

NOMENCLATURE

A_H	inner-cylinder area in contact with hexadecane
A_L	inner-cylinder area in contact with liquid
A_{tot}	total surface area of inner cylinder
A_W	inner-cylinder area in contact with water
D	diameter of inner cylinder
D_o	diameter of outer cylinder
F	Churchill–Chu Prandtl number function, equation (10)
g	acceleration of gravity
H	height of inner cylinder
H_o	height of outer cylinder
k	thermal conductivity
k_H	thermal conductivity of hexadecane
k_L	thermal conductivity of liquid
k_W	thermal conductivity of water
Nu	average inner-cylinder Nusselt number for single-fluid case, equation (1)
$Nu_{A/H}$	average inner-cylinder Nusselt number for air–hexadecane system, equation (2)
$Nu_{A/W}$	average inner-cylinder Nusselt number for air–water system, equation (2)
$Nu_{H/W}$	average inner-cylinder Nusselt number for hexadecane–water system, equation (4)
Pr	Prandtl number
Q_{bot}	inner-cylinder bottom-surface heat transfer rate
Q_{side}	inner-cylinder side-surface heat transfer rate
Q_{top}	inner-cylinder top-surface heat transfer rate
Ra	Rayleigh number, equation (8)
Ra_H	equation (8) applied to hexadecane
Ra_W	equation (8) applied to water
$Ra_{H/W}$	Rayleigh number, equation (9)
T_i	temperature of inner cylinder
T_o	temperature of outer cylinder
z^*	position of interface, Fig. 1.
Greek symbols	
β	coefficient of thermal expansion
ν	kinematic viscosity.
Subscripts	
A	air
H	hexadecane
L	liquid
W	water.

the Prandtl number, whose value ranged from about 0.71 to 40.

The literature survey presented in ref. [1] continues to apply here, and no repetition is needed. That survey demonstrated that prior work on two-fluid natural convection was both sparse and unrelated to the present work.

EXPERIMENTAL APPARATUS

The description of the experimental apparatus is facilitated by making reference to Fig. 1, which is a schematic vertical sectional view. As seen there, the inner and outer cylinders define an enclosed space filled by two fluids or, as a baseline case, by a single fluid. The inner cylinder was suspended within the outer cylinder by a fine nylon line—this means of support being employed to avoid extraneous heat losses. In turn, the outer cylinder was supported from below by a three-legged stand whose legs rested on the floor of a large (55 gal) polyethylene tank (not shown). The tank served to contain a constant-temperature water bath which totally surrounded the outer cylinder. The key dimensions of the apparatus are the diameter D and height H of the inner cylinder, both equal to 3.401 cm, and the inner diameter D_o and internal height H_o of the outer cylinder, whose common value was 15.60 cm. These dimensions yield the dimension ratio $D/D_o = 0.22$.

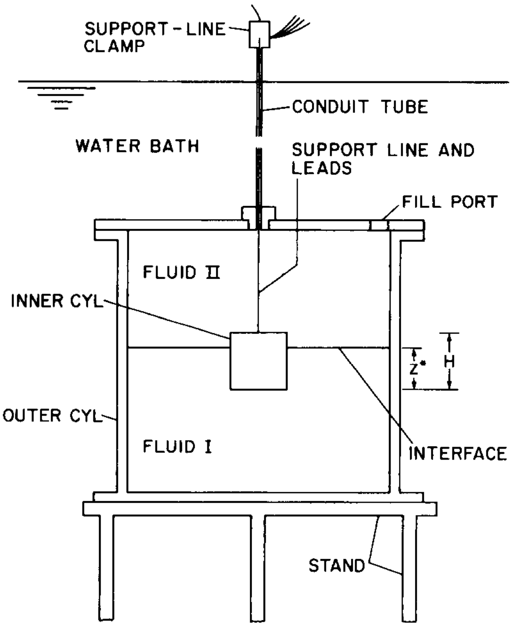

FIG. 1. Schematic vertical sectional view of the experimental apparatus.

The inner cylinder consisted of a thick-walled copper shell within which was situated a wire-wound aluminum heater core. The use of copper and of a relatively thick wall (~ 0.8 cm) was motivated by the objective of obtaining temperature uniformity. To additionally further this objective, the heater was designed with numerous (i.e. 40) uniform and closely spaced windings. Centrally positioned, fine-diameter thermocouples were installed in the upper and lower walls of the cylinder in order to detect the presence of residual temperature nonuniformities.

To facilitate the suspension of the cylinder, a 0.074-cm-diameter hole (only slightly larger than the nylon line) was drilled axially through the upper wall, precisely at its center. The nylon line passed through the hole into the bore, where it was secured. The thermocouple and heater wire leads exited the bore of the cylinder via another, slightly larger hole situated as close as possible to the nylon-line access hole.

The outer cylinder was also fabricated from copper, with walls approximately 0.6 cm thick. The upper wall of the cylinder was made readily removable to facilitate the installation and extraction of the inner cylinder and of the various working fluids. To ensure the absence of leaks, an O-ring was installed in the flange at the interface between the upper wall and the main body of the cylinder. Ten thermocouples were installed at various locations along the walls of the cylinder to measure its temperature.

At the center of the upper wall, a hole was drilled through the thickness and tapped to accommodate a pipe fitting (of brass) which, after installation, was made flush with the lower face of the wall. A 0.635-cm-diameter brass tube was then implanted in the fitting and locked in place with a leak-free seal. The tube served as a conduit to convey the nylon support line and lead wires from the inner cylinder vertically upward through the surrounding water bath and out into the air above the water surface.

The overall length of the conduit tube was 41 cm, of which the upper 4 cm protruded above the water. At its upper end, the tube terminated in a clamp which enabled the nylon line to be locked securely in place, thereby fixing the elevation at which the inner cylinder was suspended within the outer cylinder. As indicated in Fig. 1, the nylon line and the lead wires were separated at the clamp.

Also indicated in the figure is a port situated in the upper wall of the outer cylinder which was used in certain aspects of the filling of the enclosure with liquids. During the experiments proper, the port was closed by a brass pipe plug which had been machined to be flush with the lower face of the upper wall.

With regard to instrumentation, mention has already been made of the thermocouples installed in the walls of the inner and outer cylinders. In addition, two thermocouples were situated in the constant temperature water bath to monitor its spatial and temporal uniformity. All thermocouples were made from pre-calibrated, Teflon-coated 0.00762-cm-diameter chromel and constantan wire. The thermocouple e.m.f.s were read to 0.001 μV.

Power was provided to the inner cylinder heater by one of three d.c. supplies, depending on the heating rate. The uniformity of the power input during the duration of a data run was to within 0.05%. The readings of the heater voltage and current (measured as a voltage drop across a calibrated shunt) were to either four or five significant figures, depending on the power input.

In positioning the inner cylinder and in measuring the height of the fluid–fluid interface, use was made of a dial-gage-equipped caliper having a smallest scale reading of 0.001 in. and of an optical cathetometer having a resolution of 0.005 cm.

The constant-temperature bath was operated at the temperature of the thermally controlled room, except for the experiments involving hexadecane. Since hexadecane has a melting temperature of 18.8°C, it was deemed appropriate to operate at a slightly elevated temperature to achieve better accuracy in the thermophysical properties and to avoid the region of most rapid variation of the viscosity with temperature. To this end, for the hexadecane experiments, the bath was maintained at approximately 30°C.

As already noted, the fluids employed in the experiments were air, water, and hexadecane. The water was distilled water, while the hexadecane was 99% pure n-hexadecane.

EXPERIMENTAL PROCEDURE

Positioning of the inner cylinder and the interface

There were two types of measurements which were used in determining the position z^* of the fluid–fluid interface relative to the base of the inner cylinder (see Fig. 1 for the definition of z^*). One of these yielded the vertical distance between the base of the inner cylinder and the lower face of the upper wall of the outer cylinder. This distance was measured with the upper wall removed from the remainder of the outer cylinder and positioned on a stand situated on a laboratory bench, with the inner cylinder hanging below.

First, by pulling up tightly on the nylon support line, the inner cylinder was brought into contact with the upper wall of the outer cylinder. For this configuration, the cathetometer was used to read the vertical position of the base of the inner cylinder. Then, the inner cylinder was repositioned to approximate a preselected elevation, and the vertical position of the base was read once again with the cathetometer. If the difference between the aforementioned readings did not yield the desired vertical distance, the inner cylinder was further repositioned; and so on until the desired distance was achieved.

The second measurement yielded the height to which the outer cylinder was filled with the heavier of the two fluids in the respective two-fluid system, i.e. the height of the fluid which occupied the lower part of the enclosure. This measurement was made with the upper wall of the outer cylinder removed and in the absence of the inner cylinder, with air filling the space above the liquid whose height was to be measured. For the measurement, the movable bar of a caliper was fitted with an extension which terminated in a sharp needle. The caliper was mounted on a bridge-like fixture which spanned the open top of the outer cylinder, with the needle pointing vertically downward.

For a reference reading, the needle was first brought into contact with the bottom of the outer cylinder in the absence of the liquid. Then, the needle was raised to an elevated position, and the cylinder was filled with liquid approximately to mid-height. The needle was then lowered until it made contact with the liquid surface, at which point the caliper was read again. The contact was detected via an electric circuit for water and visually for hexadecane. If the difference of the readings did not yield a liquid level sufficiently close to the preselected value, liquid was either added or removed until the desired result was obtained.

From the aforementioned readings, and taking account of the volume displaced by the presence of the inner cylinder, the position z^* of the interface was calculated. The calculated value does not take account of deviations from flatness of the interfacial surface due to the possible presence of a meniscus at the inner and outer cylinders. This issue will be addressed more fully shortly. For all of the two-fluid systems, the fluid–fluid interface was situated at approximately the mid-height of the outer cylinder.

For the experiments involving only a single fluid, the mid-height of the inner cylinder was positioned approximately at the mid-height of the outer cylinder. The positioning of the inner cylinder was accomplished in conjunction with the first type of measurement described in the foregoing.

Surface preparation

Lapping compounds of various grits were used to polish the surfaces which bounded the enclosed space. In the single-fluid experiments involving air, the final polishing was performed with 1200-grit compound to give a mirror-like finish which promoted low values of radiative heat transfer. For the other cases, primary attention was given to achieving hydrodynamically smooth conditions, for which final polishing with 1000-grit compound was more than sufficient.

For the air–water system, it was found that by diligent final polishing of the inner cylinder with 1000 grit, the meniscus could be made to disappear; that is, the fluid–fluid interface was perfectly flat and perpendicular to the surface of the inner cylinder. This remarkable finding was verified both visually and with the cathetometer. Furthermore, several persons were invited to inspect the interface, and all came to the same conclusion.

Similar surface treatment did not yield flat interfaces for the air–hexadecane and hexadecane–water systems. In particular, for the air–hexadecane case, the meniscus curved upward, while for hexadecane–water the meniscus curved downward.

Setup of the experiments

The essential features of the setup of the experiments subsequent to the positioning of the inner cylinder will now be discussed, first for the single-fluid cases and then for the two-fluid cases. For the single-fluid air experiments, the setup was completed simply by putting the upper wall of the outer cylinder in place. For the single-fluid liquid experiments, either with water or hexadecane, most of the filling of the enclosure was performed with the upper wall absent. When the upper wall was put in place, the filling was continued through the port provided for this purpose. When the enclosure was ostensibly full, the apparatus was tilted so as to drive a small amount of liquid upward into the conduit tube and create an air space just below the fill port. With the tilt maintained, this space was filled with liquid and the port capped, after which the apparatus was returned to the level position.

For the two-fluid systems, the filling and height measurement of the heavier of the two fluids has already been described. In the air–water and air–hexadecane cases, no other fluid had to be added, and the setup was completed by putting the upper wall of the enclosure in place. However, strict precautions were taken to avoid extraneous contact (i.e. above the interface) of the liquid with the inner cylinder due to splashing.

For the hexadecane–water case, the upper wall of the enclosure was put in place and the inner cylinder set in its final position prior to the introduction of the hexadecane. The hexadecane was introduced into the enclosure by means of a pipe connected to the fill port. Had the filling of the hexadecane occurred prior to the final positioning of the inner cylinder, a coating of hexadecane might have been left on those portions of the cylinder that later contacted the water.

Operating procedure

The general approach to the execution of the experiments was to fix the geometry of the enclosure and to parametrically vary the temperature difference between the inner and outer cylinders. For the single-fluid cases, the geometry was fixed by positioning the mid-height of the inner cylinder at the mid-height of the outer cylinder, while for the two-fluid cases, the geometry was fixed by specifying z^*/H which, in turn, was varied parametrically. When air was used as the working fluid, the temperature difference ranged from about 1 to 100°F. When a liquid was involved, the range of the temperature difference was from about 1 to 20°F. The smaller value of the maximum temperature difference for the liquid-related experiments was selected to avoid excessive variations of the thermophysical properties.

For all cases except hexadecane–water, the temperature difference was spanned in seven steps, while five steps were used for the hexadecane–water experiments. Because of significant meniscus effects encountered for the latter two-fluid system, replicate experiments were performed for all z^*/H and temperature differences.

Prior to the initiation of a sequence of data runs at a fixed geometry, thermal equilibrium was established between the apparatus and the constant temperature bath. Then, for each data run, a waiting period was allowed (3 h was found to be more than sufficient) for steady state to be established prior to the collection of data.

DATA REDUCTION

In the presentation of the experimental results, different definitions of the average Nusselt number will be used to reflect the specifics of the heat flow paths for the various fluid configurations. Consideration need be given only to the inner-cylinder Nusselt number, since no new information is provided by presenting separate results for the outer-cylinder Nusselt number.

The electric power input to the heater core yielded, after correction, the rate of convective heat transfer from the inner cylinder as a whole. This quantity will be denoted by Q_{tot}. The corrections were for radiative transfer at the surfaces of the inner cylinder exposed to air (emissivity = 0.03).

For the single-fluid cases, the Nusselt number was defined as

$$Nu = Q_{\mathrm{tot}}D/kA_{\mathrm{tot}}(T_{\mathrm{i}}-T_{\mathrm{o}}). \tag{1}$$

In this equation, A_{tot} denotes the total surface area of the inner cylinder, and D is its diameter. The temperatures T_{i} and T_{o} are, respectively, those of the inner and outer cylinders, while the thermal conductivity k is that of the participating fluid at the reference temperature $(T_{\mathrm{i}}+T_{\mathrm{o}})/2$.

In the air–liquid systems (air–water and air–hexadecane), most of Q_{tot} is transferred from the portion of the inner cylinder that is in contact with the liquid. A Nusselt number definition which takes this into account is

$$Nu_{\mathrm{A/L}} = Q_{\mathrm{tot}}D/(kA)_{\mathrm{L}}(T_{\mathrm{i}}-T_{\mathrm{o}}). \tag{2}$$

Here, A_{L} is the surface area of the inner cylinder that contacts the liquid, i.e.

$$A_{\mathrm{L}} = \pi D^2/4+\pi Dz^* \tag{3}$$

and k_{L} is the thermal conductivity of the liquid at $(T_{\mathrm{i}}+T_{\mathrm{o}})/2$.

In the liquid–liquid system (hexadecane–water), significant heat transfer occurs at the portions of the inner cylinder that are in contact with both of the liquids. To reflect the participation of both liquids, the Nusselt number was defined as

$$Nu_{\mathrm{L/L}} = Q_{\mathrm{tot}}D/(kA)_{\mathrm{L/L}}(T_{\mathrm{i}}-T_{\mathrm{o}}) \tag{4}$$

where

$$(kA)_{\mathrm{L/L}} = (kA)_{\mathrm{W}}+(kA)_{\mathrm{H}} \tag{5}$$

$$A_{\mathrm{W}} = \pi D^2/4+\pi Dz^* \tag{6}$$

$$A_{\mathrm{H}} = \pi D^2/4+\pi D(H-z^*) \tag{7}$$

and k_{W} and k_{H} are the thermal conductivities of water and hexadecane at $(T_{\mathrm{o}}+T_{\mathrm{i}})/2$.

The Nusselt number results will be presented as a function of the Rayleigh number, for which two definitions were employed. One definition, which was applied for all the investigated cases, is

$$Ra = [gB(T_{\mathrm{i}}-T_{\mathrm{o}})D^3/\nu^2]Pr. \tag{8}$$

Here, the thermophysical properties β, ν, and Pr are those of the dominant heat transfer fluid evaluated at $(T_{\mathrm{i}}+T_{\mathrm{o}})/2$. In the single-fluid cases, the dominant fluid is the single fluid itself. In the two-fluid cases, the dominant fluid was that which occupied the lower part of the enclosure, i.e. water in the air–water and hexadecane–water systems, and hexadecane in the air–hexadecane system.

For the investigated liquid–liquid system (hexadecane–water), an alternative definition of the Rayleigh number was considered, namely

$$Ra_{\mathrm{L/L}} = (A_{\mathrm{W}}/A_{\mathrm{tot}})Ra_{\mathrm{W}}+(A_{\mathrm{H}}/A_{\mathrm{tot}})Ra_{\mathrm{H}}. \tag{9}$$

In this equation, Ra_{W} and Ra_{H} were respectively evaluated from equation (8) using water properties and hexadecane properties at $(T_{\mathrm{i}}+T_{\mathrm{o}})/2$.

SINGLE-FLUID HEAT TRANSFER RESULTS

The average inner-cylinder Nusselt numbers for the single-fluid cases are presented in Fig. 2. Experimental data are included for all three investigated fluids: air, water, and hexadecane, with nominal Prandtl numbers of 0.71, 6.2, and 40, respectively. Also appearing in the figure are a trio of lines which represent the numerical solutions of ref. [1]. In the figure, the Nusselt number is plotted as a function of the Rayleigh number evaluated in accordance with the definitions of equations (1) and (8). Note that to accommodate the different Rayleigh number ranges for air and for the liquids (water and hexadecane), lower and upper abscissa scales have been used as well as a lower and an upper ordinate scale. To enable comparison of the air data with the liquid data, the higher-Rayleigh-number air data have been plotted on the same scales as the liquid data, and the numerical predictions for air have been extended to higher Rayleigh numbers.

Examination of the figure shows that excellent agreement prevails between the experimental data and the numerical predictions. For air, the typical deviations between the experimental and numerical results are about 1.5%. For the liquids, the numerical solutions slightly overpredict the data—by 2.5–3%.

As has been observed in other natural convection problems, the use of the Rayleigh number as the independent variable does not altogether eliminate the dependence of the Nusselt number on the Prandtl

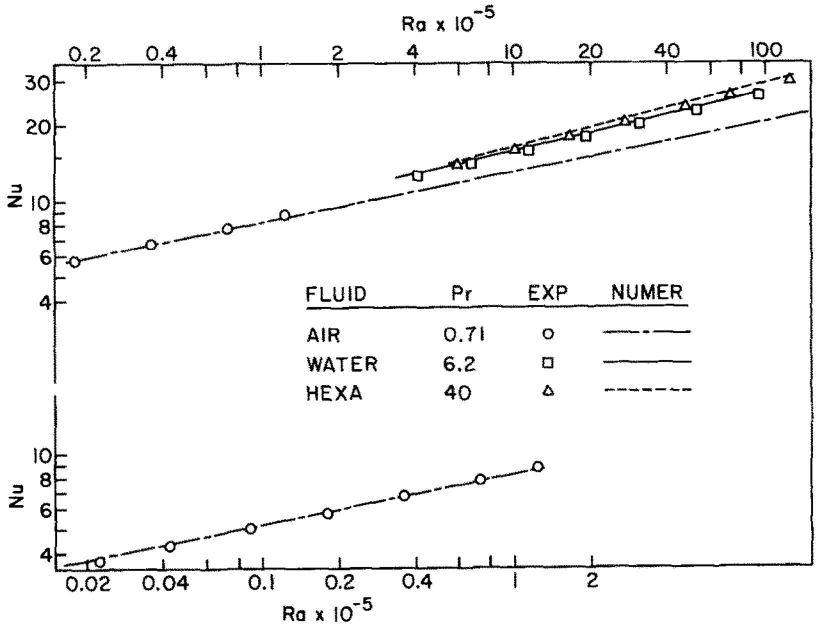

FIG. 2. Single-fluid Nusselt number results.

number. This is especially true if the Prandtl number range of interest includes values as low as ~0.7, where the sensitivity of *Nu* to *Pr* is greater than at higher Prandtl numbers. Both the experimental and numerical results display the aforementioned separate dependence on the Prandtl number, and both indicate that the slopes of the *Nu* vs *Ra* distributions are also slightly sensitive to the Prandtl number. In particular, if the numerical solutions are represented as $Nu \sim Ra^n$, the exponent n takes on values of 0.199, 0.234, and 0.253, respectively, for $Pr = 0.71$, 6.2, and 40.

With a view to bringing the data for the three fluids closer together, *Ra* is replaced as the independent variable by Ra/F, where F is a function of Prandtl number. For F, two representations were examined, one used by Churchill and Chu for correlating natural convection data for vertical plates [2] and the other (by the same authors) for correlating data for horizontal cylinders [3]. Both representations for F yielded virtually the same results, so that only one need be considered—that for the vertical plate, for concreteness. The F expression for the vertical plate correlation is

$$F = [1 + (0.492/Pr)^{9/16}]^{16/9}. \qquad (10)$$

The data of Fig. 2 have been replotted in Fig. 3 in the form of *Nu* vs Ra/F. In the new format, the results for water and hexadecane are virtually coincident. Also, the air results are now much closer to those of the liquids; however, a Rayleigh-number-dependent deviation persists. Thus, the use of the Ra/F grouping fails to yield a universal correlation when Prandtl numbers as low as ~0.7 are considered.

The air data collected here may be compared with those of ref. [4]. The test setup employed in ref. [4] was similar to that of the present experiments, except that the D/D_0 ratio was equal to 0.20 rather than 0.22 and the inner cylinder was made of aluminum rather than of copper. A comparison of the two sets of results revealed agreement within a few percent, which is quite satisfactory.

The results presented thus far were for the case in which the inner cylinder was centered in the height of the outer cylinder. Now, attention will be turned to a set of results which correspond to the mid-height of the inner cylinder situated below the mid-height of the outer cylinder, the displacement being about 0.74*D*. To motivate the downward repositioning of the inner cylinder, it is relevant to cite certain observations made from streamline and isotherm diagrams obtained from the numerical solutions.

In the lower part of the enclosure, the fluid is hydrodynamically and thermally inactive, so that little would be changed in that region by a moderate downward displacement of the inner cylinder. However, in the region above the inner cylinder, there is a vigorous plume which provides a vehicle for interaction between the top of the inner cylinder and the upper boundary of the enclosure. It was deemed worthwhile to explore whether this interaction would respond sufficiently to the repositioning of the inner cylinder to give rise to a significant change in the Nusselt number.

The experiments for the downward displaced inner cylinder were carried out with water. The resulting Nusselt numbers are presented in Fig. 4, where they are compared with the Nusselt numbers for the centered inner cylinder. The Nusselt numbers for the two cases are virtually the same, the deviations being on the order of 1.5%, with those for the displaced cylinder being higher. A similar finding was obtained from the numerical solutions. These findings suggest that the present results are applicable not only for

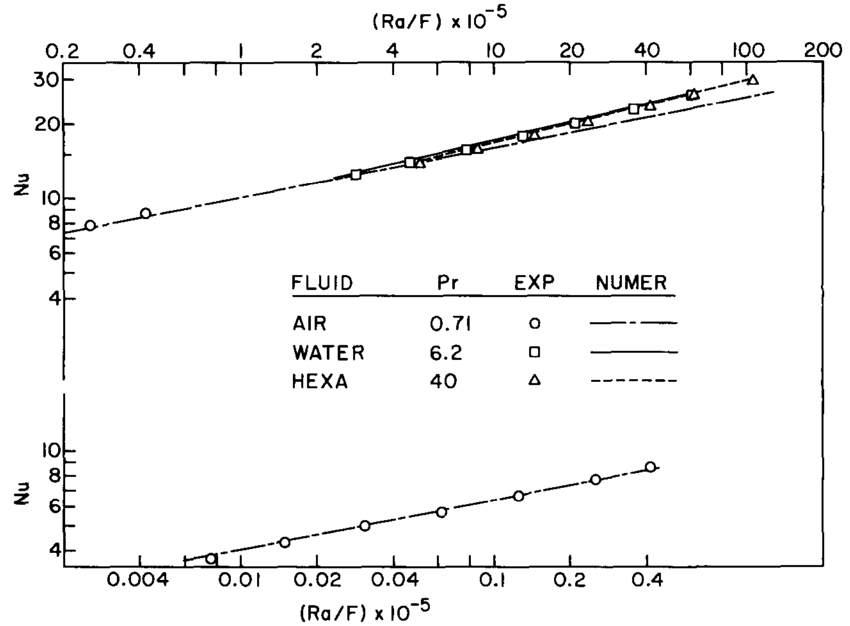

FIG. 3. Correlation of the Prandtl number effect for the single-fluid Nusselt number results.

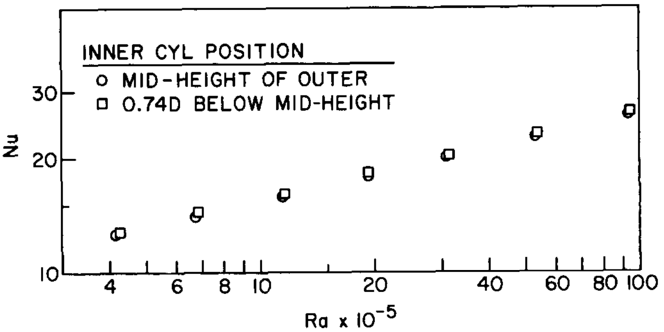

FIG. 4. Effect of the inner-cylinder vertical position on the single-fluid Nusselt number results for water.

vertically centered inner cylinders but also for inner cylinders that are moderately vertically displaced from the centered position.

The results conveyed in Figs. 2–4 pertain to the rate of heat transfer Q_{tot} from the inner cylinder as a whole. The numerical solutions also enabled the evaluation of the rates of heat transfer at the individual surfaces of the inner cylinder—the bottom, the side, and the top. These results are presented in Table 1 in the form of ratios Q_{bot}/Q_{tot}, Q_{side}/Q_{tot}, and Q_{top}/Q_{tot}. In appraising the table, it should be noted that $A_{top} = A_{bot} = A_{side}/4$.

The table shows that the side is the dominant heat transfer surface of the cylinder, a finding that is not unexpected in view of the aforementioned surface area relationships. For a more quantitative comparison, the table entries may be ratioed to yield $Q_{side}/Q_{bot} = 3.7$–4.1 for air, 4.6–5.1 for water, and 4.7–5.1 for hexadecane, showing that the heat transfer per unit area is not very different for the side and bottom surfaces. Further ratioing yields $Q_{top}/Q_{bot} = 0.43$–0.48 for air, 0.60–0.63 for water, and 0.79–1.0 for hexadecane. From this, it is seen that the bottom surface contributes more to the cylinder heat transfer rate than the top surface does, but that the contributions of the two surfaces become more equal as the Prandtl number increases.

Table 1. Heat transfer rates at the individual surfaces of the inner cylinder

Fluid	$Ra \times 10^{-5}$	Q_{bot}/Q_{tot}	Q_{side}/Q_{tot}	Q_{top}/Q_{tot}
Air	0.0169	0.194	0.713	0.093
	0.195	0.188	0.731	0.081
	1.27	0.180	0.743	0.077
Water	3.41	0.161	0.742	0.097
	15.8	0.155	0.747	0.098
	89.1	0.149	0.754	0.097
Hexadecane	5.50	0.154	0.724	0.122
	24.7	0.146	0.719	0.135
	123	0.140	0.720	0.140

In addition to the per-surface heat transfer rates displayed in Table 1, the numerical solutions provided

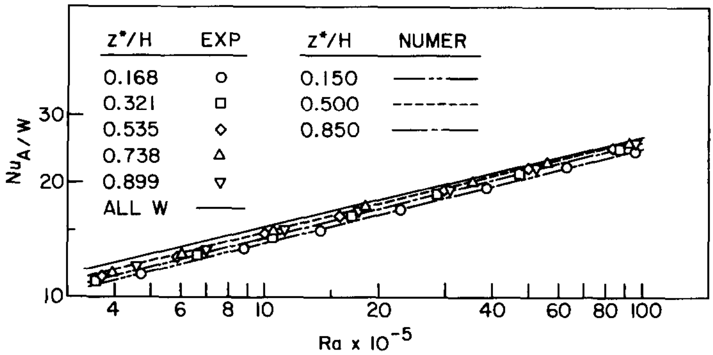

FIG. 5. Nusselt number results for the air–water system.

local heat transfer distributions along each surface. This information is available in ref. [5]. Also presented in ref. [5], but omitted here due to space limitations, are streamline and isotherm maps and velocity profiles.

TWO-FLUID HEAT TRANSFER RESULTS

The experimental data for the air–water system are brought together with the predictions of the numerical solutions in Fig. 5. The air–water Nusselt number $Nu_{A/W}$ plotted on the ordinate is that of equation (2), while the Rayleigh number on the abscissa is that of equation (8). Experimental data were collected at five interface positions characterized by $z^*/H = 0.168$, 0.321, 0.535, 0.738, and 0.899. In addition to these data, Fig. 5 also displays the single-fluid data for water. Since the latter have already been presented in Fig. 2, it is sufficient to represent them by a line in Fig. 5 and thereby avoid clutter.

Inspection of Fig. 5 shows that all the results fall in a tight band whose overall spread is about 10%. Therefore, the Rayleigh number definition of equation (2) is fairly successful in taking account of variations of the interface position. Closer inspection of the figure reveals that the data for all interface positions z^*/H between 0.321 and 0.899 lie together in a band whose spread is only about 3%. The data at the lowest investigated position $z^*/H = 0.168$ fall about 4% below the other data.

It is interesting to note that the Nusselt numbers for the water single-fluid system (represented by the solid line) lie above the two-fluid air–water Nusselt number data. With an optimal definition of the Nusselt number, the single-fluid and two-fluid data would have fallen together. The fact that the Nusselt number definition of equation (2) did not lead to such an outcome is, therefore, an imperfection. The imperfection is, fortunately, not a major one, since the deviations of the single-fluid Nusselt numbers from the middle of the two-fluid data band are only about 6% at the lower end of the Rayleigh number range and 2% at the upper end of the range.

The experimental data and numerical predictions appearing in Fig. 5 will now be compared. In this regard, it is worth noting that for the air–water system, the flat meniscus assumed in the numerical model was fully achieved in the experiments. Furthermore, the numerical results were insensitive to the nature of the force balance at the air–water interface [1, 5].

Examination of Fig. 5 reveals excellent agreement (within 2%) between the data and the numerical predictions. It is also noteworthy that the curves representing the predictions are not arranged monotonically with z^*/H. This is not surprising, since $Nu_{A/W}$ is, in essence, the ratio of Q_{tot} to A_W, both of which increase with z^*/H.

The Nusselt number results for the air–hexadecane system will now be considered. In Fig. 6, $Nu_{A/H}$ is plotted vs Ra, where these quantities are defined by equations (2) and (8), respectively. Experimental data are presented for six interface positions between $z^*/H = 0.137$ and 0.894 as well as for the hexadecane single-fluid case (solid line). Predictions from the numerical solutions corresponding to $z^*/H = 0.150$, 0.500, and 0.850 are also included in the figure.

From the figure, it is seen that the two-fluid Nusselt number data for all the investigated interface positions cluster together, indicating an insensitivity to z^*/H. The data spread is on the order of 4%. Thus, the definition used for $Nu_{A/H}$ is quite successful in bringing together the data for the various z^*/H. However, the Nusselt numbers for the hexadecane single-fluid case do not overlap with the two-fluid data but rather tend to lie high—the gap being about 5% between the single-fluid line and the middle of the two-fluid data. On the whole, the experimentally determined Nusselt numbers for the air–hexadecane system are more compactly deployed than those for the air–water system presented in Fig. 5 (also taking account of the data for the respective single-fluid systems).

With regard to the comparison between computation and experiment, note should be taken of the upward sloping meniscus, typically with a rise of 0.17 cm, at the intersection of the air–hexadecane interface and the inner cylinder. Despite the fact that the numerical model did not take account of such a meniscus, the agreement between the numerical and experimental results is very good. Not unexpectedly, the

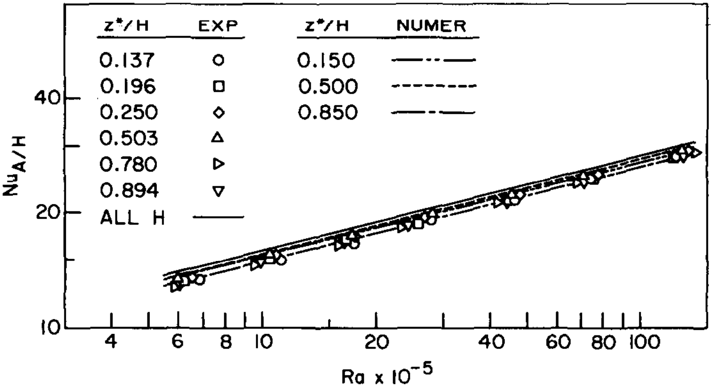

FIG. 6. Nusselt number results for the air–hexadecane system.

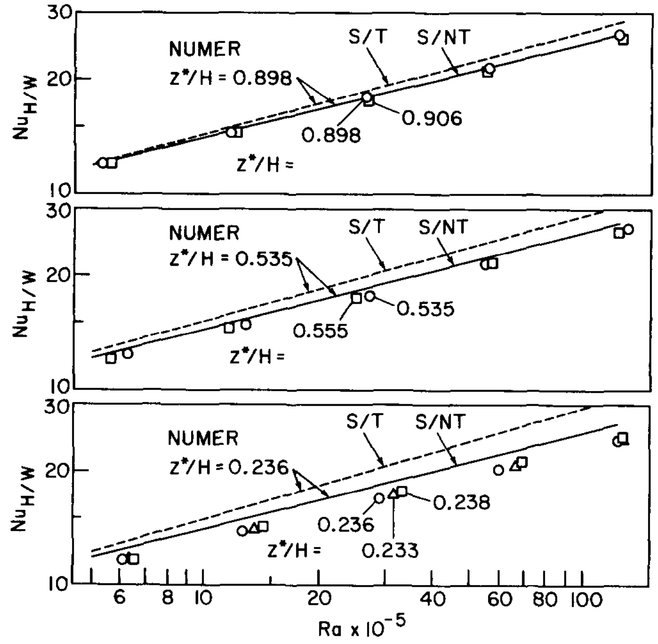

FIG. 7. Nusselt number results for the hexadecane–water system for various interface positions.

greatest deviations (about 5%) occur at the smallest z^*/H value, where the presence of the meniscus introduces the greatest uncertainty into the contact area between the hexadecane and the inner cylinder. For the other z^*/H, the agreement is in the 1–2% range. Again, as was true for the air–water system, the specifics of the interface force balance (accounting or non-accounting of interfacial tension) did not affect the predicted results for the air–hexadecane system [1, 5].

Attention is now turned to the hexadecane–water system, which is the one liquid–liquid system investigated here. In the experiments, a downward sloping meniscus existed at the intersection of the hexadecane–water interface and the inner cylinder. Typically, the downward drop of the meniscus was about 0.38 cm. Because of this, the smallest z^*/H for the hexadecane–water system ($\sim$0.24) was chosen to be greater than the smallest z^*/H for the air–water and air–hexadecane systems, neither of which had a downward sloping meniscus. Another action performed in response to the substantial meniscus-related departure of the interface from flatness was to carry out replicate experiments in order to establish the reproducibility of the data. For each set of replicate experiments, the apparatus was set up anew.

The numerically predicted Nusselt numbers for the hexadecane–water system were found to be somewhat sensitive to the nature of the force balance at the fluid–fluid interface [1, 5]. Two force balance models were considered for this system. In one model, to be designated as the *S*/*T* model, full account was taken of the interfacial shear stress (*S*) and of the interfacial tension (*T*). In the other model, the *S*/*NT* model, the shear stress was taken into account but the interfacial tension was neglected (no tension). However, in neither model were the meniscus-related departures from a flat interface considered.

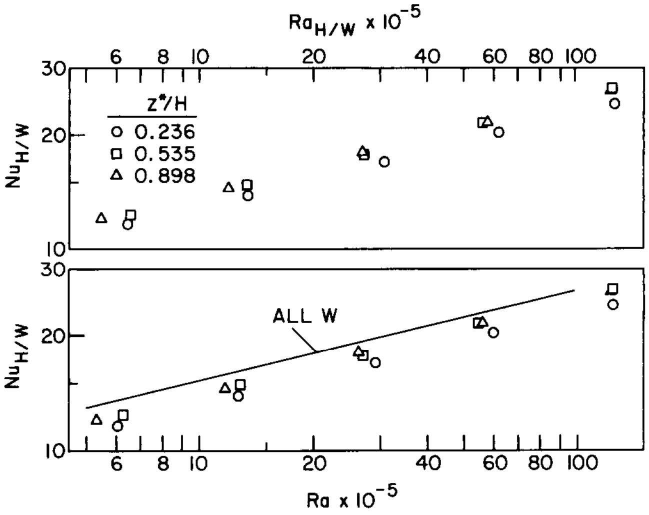

FIG. 8. Nusselt number results for the hexadecane–water system.

With the foregoing paragraphs as background, the results will now be presented, and Fig. 7 has been prepared for this purpose. The figure consists of three graphs which respectively pertain to $z^*/H \sim 0.24$, 0.54, and 0.90. Each graph contains either two or three independent sets of data and, in addition, two lines which represent the predictions of the numerical solutions. The lines respectively correspond to the S/T and S/NT models of the interface force balance which were described in the preceding paragraph. The Nusselt number $Nu_{H/W}$ which appears on the ordinate is the liquid–liquid Nusselt number defined by equation (4), while the Rayleigh number on the abscissa is that of equation (8). Note that the thermophysical properties in the Rayleigh number are those of water, since it is the dominant heat transfer fluid in the hexadecane–water system.

From Fig. 7, it is clear that the data for each z^*/H are highly reproducible, indicating that the shape of the meniscus is also reproducible.

Further inspection of the figure shows that the predictions based on the S/T model lie above those for the S/NT model, while the experimental data fall slightly below the S/NT-based predictions. To provide perspective for these findings, it may be noted that the S/NT model neglects both the interfacial tension and the meniscus. Accounting for the interfacial tension, as in the S/T model, increases the rate of heat transfer. However, when both the interfacial tension and the meniscus are taken into account, as in the experimental data, the heat transfer is slightly less than when these effects are both neglected. Thus, the meniscus effects more than neutralize the interfacial tension effects.

Because of the just-discussed neutralization of conflicting effects, the S/NT model yields fairly good predictions of the Nusselt number. The predictions are accurate to 10% or better for $z^*/H = 0.236$, 5% or better for $z^*/H = 0.535$, and about 1% for $z^*/H = 0.898$.

The effects discussed in the penultimate paragraph are most significant at low interface positions (i.e. low z^*/H) but become less important as z^*/H increases. To explain this trend, it may be noted that water is the dominant heat transfer fluid in the system, and when there is a relatively small contact area between the water and the cylinder, the overall rate of heat transfer is small. These low rates of heat transfer are readily altered by interface-related effects. However, at larger z^*/H, the heat transfer rates are considerably greater and are, thereby, less susceptible to happenings at the interface. Note that since the meniscus slopes downward, the water–cylinder contact area for a given z^*/H is actually smaller in the experimental setup than in the numerical model.

The experimental data for the various z^*/H interface positions of Fig. 7 are brought together in Fig. 8. In the lower graph of Fig. 8, the Nusselt and Rayleigh number definitions are the same as those of Fig. 7. Also plotted in that graph is a line representing the water single-fluid Nusselt number results. In the upper graph of Fig. 8, the liquid–liquid Rayleigh number of equation (9), now designated as $Ra_{H/W}$, is used as the abscissa variable.

From an inspection of Fig. 8, it is seen that the $Nu_{H/W}$ vs $Ra_{H/W}$ plot is virtually identical to the $Nu_{H/W}$ vs Ra plot. This is because, by chance, for given values of T_i and T_o, the Rayleigh numbers Ra_W and Ra_H that appear in equation (9) are virtually equal in magnitude. Because of this, it is not possible to make a definitive judgement about the relative merits of Ra and $Ra_{H/W}$ as correlation parameters for the $Nu_{H/W}$ results.

In view of the foregoing, it is sufficient to focus attention on the lower graph of Fig. 8. There, it can be seen that the data for $z^*/H = 0.535$ and 0.898 fall together, thereby supporting the definition of $Nu_{H/W}$. The $z^*/H = 0.236$ data fall 5–8% below the others, probably because the actual contact area between the water and the inner cylinder is significantly less than the nominal contact area. The water single-fluid Nusselt numbers exceed the hexadecane–water Nusselt numbers, a pattern that also occurred for the other two-fluid systems. In particular, there is a 5–9% gap between the single-fluid line and the two-fluid data for the upper two z^*/H positions.

The numerical solutions for the two-fluid systems provided streamline and isotherm maps, velocity and temperature profiles, distributions of the local heat flux, rates of heat transfer to the individual fluids, and rates of heat transfer at the individual surfaces of the cylinder. This information is, in part, presented in ref. [1], and the remainder is available in ref. [5].

CONCLUDING REMARKS

Experimental data presented here for three two-fluid natural convection systems agreed well with predictions obtained from numerical solutions performed in an antecedent study. The natural convection took place in the fluid-filled, enclosed space between two concentrically positioned vertical cylinders having different finite heights. The two-fluid pairs included air–water, air–hexadecane, and hexadecane–water. Parametric variations were made of the height of the fluid–fluid interface and of the Rayleigh number.

The agreement between the experimentally and numerically determined Nusselt numbers was especially good for the air–water system, where the model adopted for the numerical work faithfully mirrored the experimental conditions. In particular, there was no observed meniscus at the intersection of the air–water interface and the inner cylinder, which matched the assumption made in the model. For the air–hexadecane system, this assumption was not fulfilled because an upward sloping meniscus with a moderate rise was present in the experiments. Notwithstanding this, good agreement prevailed between the Nusselt numbers. In the case of the hexadecane–water system (the only liquid–liquid system investigated here), the meniscus sloped downward and gave rise to an appreciable departure from the assumed flat interface. It was found, however, that a model which neglected both the meniscus and the interfacial tension yielded Nusselt numbers in satisfactory agreement with the experimental data.

Experiments were also performed for single-fluid systems in which the respective working fluids were air, water, and hexadecane. For all the investigated fluids, excellent agreement prevailed between the Nusselt numbers from experiment and from computation. A special focus of the single-fluid work was to attempt to correlate the effect of the Prandtl number, which ranged from 0.71 to 40. For this purpose, the Churchill–Chu factor was used. This factor brought together the results for water and hexadecane ($Pr = 6.2$ and 40) but the air data ($Pr = 0.71$) remained somewhat separate.

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EXPERIENCES DE CONVECTION NATURELLE DANS DES ESPACES CLOS ET COMPLEXES CONTENANT SOIT DEUX FLUIDES SOIT UN SEUL FLUIDE

Résumé—Des coefficients de transfert thermique sont déterminés expérimentalement pour la convection naturelle dans des espaces clos entre deux cylindres concentriques verticaux ayant différentes hauteurs finies. L'espace est rempli par deux couches de fluides, l'une au-dessus de l'autre. Les systèmes binaires étudies sont air–eau, air–hexadecane et hexadecane–eau. Des expériences concernent aussi le cas où l'espace entre cylindres est rempli par un seul fluide, air, eau ou hexadecane. Des variations paramétriques sont faites pour la hauteur de l'interface entre couches et pour le nombre de Rayleigh. Des comparaisons sont faites entre les résultats expérimentaux et les prévisions à partir de la résolution numérique du problème. Un très bon accord est constaté pour les cas de fluide unique et aussi pour les cas à deux fluides lorsque les effets de ménisque et de tension interfaciale sont soit négligeables, soit petits. Lorsque ces effets sont plus importants, comme pour le système hexadecane–eau (le seul système liquide–liquide étudié), on peut améliorer l'accord en utilisant un modèle d'interface relativement simple. Pour les résultats avec fluide unique, l'effet de la variation du nombre de Prandtl (entre 0,71 et 40) est mis en lumière.

EXPERIMENTELLE UNTERSUCHUNG DER NATÜRLICHEN KONVEKTION IN HOHLRÄUMEN, DIE EIN ODER ZWEI FLUIDE ENTHALTEN

Zusammenfassung—Die Wärmeübergangskoeffizienten für natürliche Konvektion im abgeschlossenen Raum zwischen zwei konzentrischen senkrechten Zylindern unterschiedlicher endlicher Höhe wurden experimentell ermittelt. Der Innenraum war mit zwei übereinanderliegenden Fluidschichten gefüllt. Die untersuchten Zwei-Fluid-Systeme waren Luft und Wasser, Luft und Hexadekan sowie Hexadekan und Wasser. Experimente mit nur einem Fluid wurden ebenfalls durchgeführt (Luft, Wasser und Hexadekan). Es wurden Parametervariationen für die Höhe der Grenzfläche zwischen den beiden Fluiden und für die Rayleigh-Zahl durchgeführt. Es wurden Vergleiche zwischen den experimentellen Ergebnissen und den numerischen Lösungen des Problems gezogen. Sehr gute Übereinstimmung ergab sich für den Fall von nur einem Fluid und auch für den zweier Fluide, wenn Meniskus- und Grenzflächenspannungs-Einflüsse klein oder vernachlässigbar waren. Wenn diese Einflüsse bedeutsamer waren, wie für das Hexadekan–Wasser-System (das einzige untersuchte System mit zwei Flüssigkeiten), läßt sich eine befriedigende Übereinstimmung mit einem relativ einfachen Grenzflächenmodell erreichen. Bei den Ergebnissen des Ein-Fluid-Falls wurde der Einfluß der Prandtl-Zahl (zwischen 0,71 und 40) hervorgehoben.

ЕСТЕСТВЕННАЯ КОНВЕКЦИЯ В ЗАМКНУТЫХ ПОЛОСТЯХ СЛОЖНОЙ ФОРМЫ, ЗАПОЛНЕННЫХ ДВУМЯ ИЛИ ОДНОЙ ЖИДКОСТЯМИ

Аннотация—Экспериментально определены коэффициенты теплообмена при естественной конвекции в замкнутой полости между двумя концентрическими вертикальными цилиндрами различной высоты. Эта полость заполнялась двумя различными жидкостями—слой одной располагался над слоем другой. Исследовались двухжидкостные системы: воздух и вода, воздух и гексадекан, гексадекан и вода. Проводились также эксперименты при заполнении зазора между цилиндрами одной жидкостью—воздухом, водой или гексадеканом. Изучалась роль высоты границы жидкость–жидкость и числа Рэлея. Экспериментальные результаты сравнивались с расчетами, полученными из численных решений. Хорошее соответствие получено для случая одной жидкости, а также для двух жидкостей, когда мениск и поверхностное натяжение либо отсутствовали, либо были пренебрежимо малыми. В случае, когда поверхностные эффекты становятся существенными (в системе гексадекан–вода—единственной из исследованных систем жидкость–жидкость) можно достичь удовлетворительного соответствия, если использовать относительно простую модель межфазной границы раздела. Результаты, полученные для одножидкостной системы, сильно зависели от величины числа Прандтля (от 0,71 до 40).